Notes on Simple Harmonic Motion

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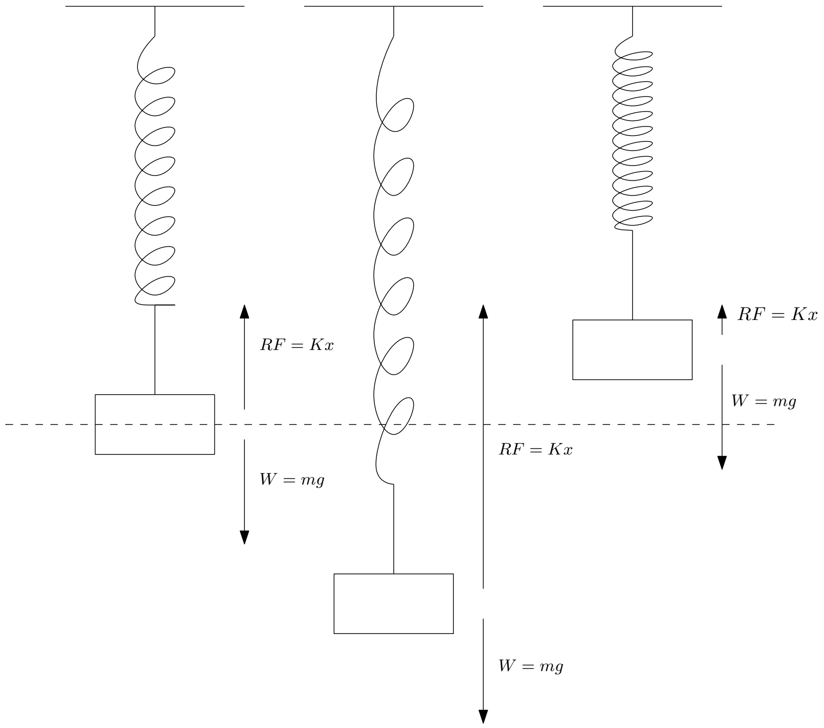
1 Introduction

SHM can be seen throughout nature from the vibration of atoms to the variability of giant stars. In between the motion of guitar strings or a child on a swing are examples of SHM.

2 Theory and Definition

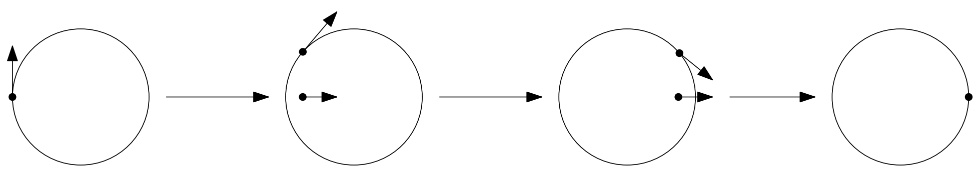
The definition of simple harmonic motion is the periodic motion whose accelerating force is proportional to its displacement and in the opposite direction.



The restoring force to the motion is given by the following equation:

$$F = -kx \tag{1}$$

Where k is a constant that depends on the particular oscillating system, and x is the displacement, thus showing that the restoring force is proportional to its displacement. The negative sign shows that the restoring force is in the opposite direction. The displacement of an object moving in simple harmonic motion is modelled with the horizontal component of circular motion as shown below.



Because simple harmonic motion can be described as the component of circular motion, we can mathematically describe the displacement of such a particle as:

$$x = Asin\omega t \tag{2}$$

Where x is the displacement of the particle relative to the equilibrium point. A is the amplitude of the oscillation. t is the time passed, and ω is the angular velocity which is given as the following equation:

$$\omega = 2\pi f \tag{3}$$

Differentiating displacement will give you an expression for the velocity of the particle at a particular time:

$$v = \dot{x} = A\omega cos\omega t \tag{4}$$

Differentiating velocity will give you an expression for the acceleration of the particle at a particular time:

$$a = \ddot{x} = -A\omega^2 sin\omega t \tag{5}$$

Substituting equation 2 into equation 5 you get the following:

$$a = -\omega^2 x \tag{6}$$

Using equation (1) and the well known formula corresponding to Newton's second law of motion($F = ma$), and substituting equation (6) for a, we get the following:

$$-m\omega^2 x = -kx$$

Where we can cancel the x terms, and divide the entire expresiion by -1:

$$k = m\omega^2 \tag{7}$$

This equation be easily used to find an expression for frequency, and inversely, time period.