Nanyang Business School

BU8201 Business Finance

Tutorial 2: Time Value of Money

(Common Questions)

Note to students: All students should learn to write down the numerical working for TVM and TVM-related questions instead of just writing down the calculator steps. In the exams, students are required to write down the numerical working. For example, when calculating FV, students should write FV = $1000(1+0.05)^{10}$*. Students will be penalized in the exams if they write "N = 10, I/YR = 5, PMT = 0, PV = 1000. Find FV".*

1) **Non-annual Compounding.** One year from today you must make a payment of $10,000. To prepare for this payment, you plan to make two equal quarterly deposits (at the end of Quarters 1 and 2) in a bank that pays 4% nominal interest compounded quarterly. How large must each of the two payments be?

2) **Evaluating lump sums and annuities**. Crissie just won the lottery, and she must choose between three award options. She can elect to receive a lump sum today of $61 million, to receive 10 end-of-year payments of $9.5 million, or 30 end-of-year payments of $5.5 million.

a) If she thinks she can earn 7 percent annually, which should she choose?

b) If she expects to earn 8 percent annually, which is the best choice?

c) If she expects to earn 9 percent annually, which would you recommend?

d) Explain how interest rates influence the optimal choice.

3) **Effective versus nominal interest rates.** Bank A pays 4% interest compounded annually on deposits, Bank B pays 3.75% compounded semiannually, and Bank C pays 3.5% compounded daily.

a) Which bank would you use? Why?

b) If you deposited $5,000 in each bank today, how much would you have at the end of 2 years?

c) What nominal rate would cause Banks B and C to provide the same effective annual rate as Bank A?

d) Suppose you do not have $5000 now but need it at the end of 1 year. You plan to make a series of deposits, annually for Bank A, semiannually for Bank B, and daily for Bank C, with payments beginning today. How large must the payments be to each bank?

4) **Interest portions and remaining balance**. The Jackson family is interested in buying a home. The family is applying for a $150,000, 30-year mortgage. Under the terms of the mortgage, they will receive $150,000 today to help purchase their home. The loan will be fully amortized over the next 30 years. Current mortgage rates are 8 percent. Interest is compounded monthly and all payments are due at the end of the month.

a) What is the monthly mortgage payment?

b) What will be the remaining balance on the mortgage after the first year?

5) **Required annuity payments.** A father is now planning a savings program to put his daughter through college. She just celebrated her 13^{th} birthday, she plans to enroll at the university in 5 years when she turns 18 years old, and she should graduate in 4 years. Currently, the annual cost (for everything – food, clothing, tuition, books, transportation, and so forth) is $15,000, but these costs are expected to increase by 5% annually. The college requires that this amount be paid at the start of the school year. She now has $7,500 in a college savings account that pays 6% annually.

How large must each payment be if the father makes five equal annual deposits into her account; the first deposit today and the fifth deposit on the daughter's 17^{th} birthday? [Hint: Calculate the cost (inflated at 5%) for each year of college and find the PV of these costs, discounted at 6%, as of the day she enters college. Then find the compounded value of her initial $7,500 on that same day. The difference between the PV costs and the amount that would be in the savings account must be made up by the father's deposits].

Self-practice question

Terry started a savings plan some years ago when he was 30 years old, and the savings plan would allow him to accumulate $1,000,000 in his bank account to meet his early retirement expenses by the time he reaches 50 years old. The savings are made at the end of every month, and the bank pays a nominal annual interest rate of 6%, compounded monthly.

(a) What is Terry's monthly contribution to his savings account under this plan, assuming the last payment was made when he turns 50 years old?

(b) Terry has just turned 40 today and he just made his 120^{th} contribution. An unfortunate economic crisis has caused him to lose his job, and he is temporarily unable to continue the monthly contributions. In fact, he would need to make monthly withdrawal of $1,500 from the savings account to meet living expenses, starting the end of the month. Assuming Terry is out of job for 3 years till he turns 43 years old (i.e. he would have to make 36 monthly withdrawals), how much money will he have in his savings account by the time he reaches 43 years old?

(c) At age 43, Terry finds a new job with higher salary than before, he intends to resume his savings plan but with higher monthly contributions so as to reach the same $1,000,000 that he set out to achieve by time he turns 50. The new monthly contributions are also made at the end of every month. What should the new higher monthly contribution be?

Answers to self-practice question

Note: There are many ways to solve the questions. The below presentation is only one of the many ways.

(a)

$1{,}000{,}000 = \text{PMT}^*(1+6\%/12)^{239} + \ldots. + \text{PMT}^*(1+6\%/12)^{1} + \text{PMT}$

PMT (monthly installment) = **$2,164.31**

Alternatively,

$1{,}000{,}000 = \text{PMT}\left[\frac{(1+\frac{0.06}{12})^{240}-1}{0.06/12}\right]$

PMT = **$2,164.31**

(b)

Step 1 - Compute the amount he has when he reaches 40 years old (t=120)

$FV_{120} = 2{,}164.31(1+6\%/12)^{119} + 2{,}164.31(1+6\%/12)^{118} + \ldots + 2{,}164.31$

= $354,685.80

Step 2 – Compound this amount to age 43 (t=156)

$FV_{156(s)} = 354{,}685.80(1+6\%/12)^{36}$

= $424,445.60

Step 3 – Find the FV (t=156) of the 36 monthly withdrawal of $1,500

$FV_{156(w)} = 1{,}500(1+6\%/12)^{35} + 1{,}500(1+6\%/12)^{34} + \ldots + 1{,}500$

= $59,004.16

Step 4 – Find the balanced amount he has when he reaches 43 years old (t=156)

Amount = 424,445.60 – 59,004.16 = **$365,441.44**

(c)

Step 1 – Find the FV of \$365,441.44 at age 50 (t=240)

$FV_{240} = 365{,}441.44(1+6\%/12)^{240\text{-}156}$

= \$555,606.07

Step 2 – Compute the extra amount in order to reach \$1,000,000 at age 50 (t=240)

Extra amount = 1,000,000 – 555,606.07 = \$444,393.93

Step 3 – Compute the monthly installment required between age 43 and age 50

$444{,}393.93 = PMT(1+6\%/12)^{83} + PMT(1+6\%/12)^{82} + \ldots + PMT(1+6\%/12) + PMT$

PMT (new monthly contribution) = **\$4,269.98**