

NANYANG TECHNOLOGICAL UNIVERSITY
SPMS/DIVISION OF MATHEMATICAL SCIENCES

2020/21 Semester 1

MH1100 Calculus I

Homework 1, 25 Sept.

Due by 6 pm, 6 October 2020; Hard copy is preferred.

Question 1

(4 marks)

- (a) Use ϵ - δ definition to prove that

$$\lim_{x \rightarrow 1} \frac{x^2 + x - 2}{x - 1} = 3.$$

- (b) Use definition to prove that

$$\lim_{x \rightarrow -1^+} \frac{1}{x + 1} = \infty.$$

Question 2

(4 marks)

Find the limits if exist.

(a) $\lim_{x \rightarrow 0} \frac{x^2 - 3x}{\tan(2x)}$

(b) $\lim_{x \rightarrow 0} \frac{\frac{1}{x-4} + \frac{1}{x+4}}{4x}$

(c) $\lim_{x \rightarrow 4} \sec\left(\frac{x-4}{\sqrt{x}-2}\pi\right)$

(d) $\lim_{t \rightarrow 0} \frac{2t}{\sqrt{2+t} - \sqrt{2-t}}$

(e) $\lim_{x \rightarrow 1} \frac{\sqrt{3x+6}-3}{x^2-1}$

(f) $\lim_{h \rightarrow 0} \frac{(2+h)^3 - 8}{h}$

(g) $\lim_{h \rightarrow 0} \frac{\frac{1}{x^2} - \frac{1}{(x-h)^2}}{h}$

(h) If $\lim_{x \rightarrow 1} \frac{x-1}{2f(x)+4} = 3$, find $\lim_{x \rightarrow 1^+} f(x)$.

Question 3

(4 marks)

Let $f(x) = x^3 + \sqrt{2x+1} - 5$.

- (a) Find the domain of $f(x)$.
- (b) Use the definition of continuity to show that $f(x)$ is continuous on its domain.
- (c) Use the definition to find the derivative function $f'(x)$.
- (d) Prove that the equation $f(x) = 0$ has a root in its domain.

Question 4

(4 marks)

Find the values of a , b , c , and d that make f differentiable everywhere.

$$f(x) = \begin{cases} \frac{x^2-4}{x-2}, & \text{if } x < 2; \\ ax^2 + bx + 6, & \text{if } 2 \leq x < c; \\ 2x + d, & \text{if } x \geq c. \end{cases}$$

Question 5

(4 marks)

Find the derivatives of the functions in Part (a)-(d) and evaluate the derivative in Part (e).

$$\begin{array}{lll} \text{(a) } y = (x^6 + 2x^3)^4 & \text{(b) } y = \left(\frac{x^4 + 1}{2x^2 + 1} \right)^5 & \text{(c) } y = \sqrt{\frac{1 + \cos t}{1 + \sin t}} \\ \text{(d) } y = [x + (x + \tan^2 x)^3]^4 & \text{(e) } D^{203} \cos(2x) & \end{array}$$