NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2020/21 Semester 1 MH1100 Calculus I Tutorial 1, Week 2

Your tutor will aim to discuss: Problem 1, 4, 5, 6, and 10

Problem 1 What is the domain of the function

$$f(x) = \frac{1}{\sqrt{3} + 2\cos x}.$$

Problem 2 Show that if f is both even and odd, then $f(x) = 0$ for every x in the domain of f.

Problem 3 Let $f(x)$ be defined on the close interval $[-a, a]$, prove that

(a) $F(x) = f(x) + f(-x)$, $x \in [-a, a]$ is even;
(b) $G(x) = f(x) - f(-x)$, $x \in [-a, a]$ is odd;
(c) $f(x)$ can be expressed as a sum of an even function and an odd function.

Problem 4 Show that the function $f(x)$ below is odd,

$$f(x) = \ln\left(x + \sqrt{1 + x^2}\right).$$

Problem 5 Find the domain of $f(x) = \frac{|x|}{x^2}$, sketch the function, and find the interval where f is an increasing function.

Problem 6 Prove that $f(x) = \frac{x}{1-x}$ is increasing on the interval $(-\infty, 1)$.

Problem 7 Starting with the graph of $y = 1/x$, use geometric transformations to plot the graph of the function

$$f(x) = \frac{3}{x - 1}.$$

Problem 8 Given $f(x) = x^3 + 2x^2$ and $g(x) = 3x^2 - 1$, find (a) $f + g$, (b) $f - g$, (c) fg, (d) f/g and state their domains.

Problem 9 A spherical balloon is being inflated and the radius of the balloon is increasing at a rate of 2 cm/s.

(a) Express the radius r of the balloon as a function of the time t (in seconds).
(b) If V is the volume of the balloon as a function of the radius, find $V \circ r$ and interpret it.

Problem 10 For any given real numbers a and b, prove that $|a| - |b| \leq |a \pm b| \leq |a| + |b|$.