NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2020/21 Semester 1 MH1100 Calculus I Tutorial 2, Week 3

Your tutor will aim to discuss: Problem 1, 2, 5, 6, 9, and 10

Problem 1

(a) Suppose $g(x)$ is an even function and we set $h = f \circ g$, where f is some other function. Is $h(x)$ even?
(b) Suppose $g(x)$ is an odd function and we set $h = f \circ g$, where f is some other function. Is $h(x)$ always odd?
(c) Suppose $g(x)$ is an odd function and we set $h = f \circ g$, where f is odd. What about $h(x)$?
(d) Suppose $g(x)$ is an odd function and we set $h = f \circ g$, where f is even. What about $h(x)$?

Problem 2 Let $f(x) = ax + b$ and $g(x) = cx + d$. What condition must be satisfied by the constants a, b, c, d in order that $(f \circ g)(x) = (g \circ f)(x)$ for every value of x?

Problem 3 The point $P(1, 1)$ lies on the the graph of the function $f(x) = \frac{3x}{1+2x}$.

(a) If Q is the point $\left(x, \frac{3x}{1+2x}\right)$, use your calculator to find the slope of the secant line PQ for the values of x: (i) 0.5; (ii) 0.9; (iii) 0.99; (iv) 0.999; (v) 1.5; (vi) 1.1; (vii) 1.01; and (viii) 1.001.

(b) Using the results of part (a), guess the value of the slope of the tangent line to the curve at $P(1, 1)$.

(c) Using the slope from part (b), find an equation of the tangent line to the curve at $P(1, 1)$.

Problem 4 The displacement of a particle moving back and forth along a straight line is given by the equation of motion $s = 2\sin \pi t + 3\cos \pi t$, where t is measured in seconds.

(a) Find the average velocity for during the time period : (i) $[1, 2]$; (ii) $[1, 1.1]$; (iii) $[1, 1.01]$; (iv) $[1, 1.001]$.

(b) Estimate the instantaneous velocity of the particle when $t = 1.0$.

Problem 5 Consider a function $h(x)$ with the graph as shown in Figure 1. State the value of the following quantity, if it exists. If it does not exist, explain why.

(a) $\lim_{x\to -3^-} h(x)$	(b) $\lim_{x\to -3^+} h(x)$	(c) $\lim_{x\to -3} h(x)$	(d) $h(-3)$
(e) $\lim_{x\to 0^-} h(x)$	(f) $\lim_{x\to 0^+} h(x)$	(g) $\lim_{x\to 0} h(x)$	(h) $h(0)$
(i) $\lim_{x\to 2} h(x)$	(j) $h(2)$	(k) $\lim_{x\to 5^+} h(x)$	(l) $\lim_{x\to 5^-} h(x)$

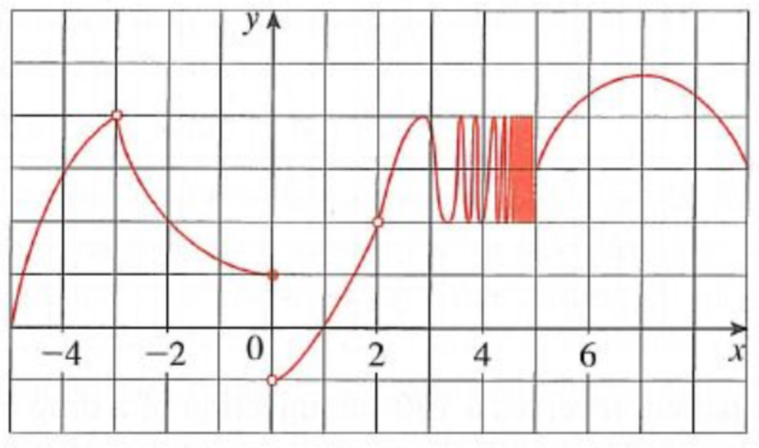

FIGURE 1. Graph of $h(x)$ in Problem 5

Problem 6 Give an example of a function f satisfying the following conditions: $\lim_{x\to 0^-} f(x) = 1$, $\lim_{x\to 0^+} f(x) = -1$, $\lim_{x\to 2^-} f(x) = 0$, $\lim_{x\to 2^+} f(x) = 1$, $f(2) = 1$, and $f(0)$ is undefined.

Problem 7 Guess the value of the limit

$$\lim_{x\to 3} \frac{x^2 - 3x}{x^2 - 2x - 3}$$

by evaluating the function at the points

$$x = 3.5, 3.1, 3.05, 3.01, 3.005, 2.5, 2.9, 2.95, 2.99, 2.995.$$

Problem 8

(a) Evaluate the function $f(x) = x^2 - (2^x/1000)$ for $x = 1$, 0.8, 0.6, 0.4, 0.2, 0.1, and 0.05, and guess the value of

$$\lim_{x\to 0} \left(x^2 - \frac{2^x}{1000} \right).$$

(b) Evaluate $f(x)$ for $x = 0.04$, 0.02, 0.01, 0.005, 0.003, and 0.001. Guess again.

Problem 9 Consider the function $f(x) = \tan \frac{\pi}{x}$.

(a) Show that $f(x) = 0$ for $x = \frac{1}{1}, \frac{1}{2}, \frac{1}{3}, \cdots, \frac{1}{n}, \cdots$.

(b) Show that $f(x) = 1$ for $x = \frac{4}{1}, \frac{4}{5}, \frac{4}{9}, \cdots, \frac{4}{4n-3}, \cdots$.

(c) What can you conclude about $\lim_{x\to 0^+} \tan \frac{\pi}{x}$?

Problem 10

(a) Use numerical and graphical evidence to guess the value of the limit

$$\lim_{x\to 4} \frac{x^3 - 64}{\sqrt{x} - 2}.$$

(b) How close to 4 does x have to be to ensure that the function in part (a) is within a distance 2.0 of its limit?