NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2020/21 Semester 1 MH1100 Calculus I Tutorial 3, Week 4

Your tutor will aim to discuss: Problem 3, 4, 7, 10, 13, and 14

Problem 1 Determine the infinite limit (show the sign of the infinite limit).

$$\lim_{x\to 5^+}\frac{x+1}{x-5}, \qquad \lim_{x\to 5^-}\frac{x+1}{x-5}, \qquad \lim_{x\to 1}\frac{2-x}{(x-1)^2}, \qquad \lim_{x\to 3^-}\frac{\sqrt{x}}{(x-3)^5},$$

$$\lim_{x\to -3^+}\frac{x-2}{x^2(x+3)}, \qquad \lim_{x\to 0}\frac{x-1}{x^2(x+2)}, \qquad \lim_{x\to (\pi/2)^+}\frac{\sec x}{x}, \qquad \lim_{x\to \pi^-}\cot x$$

Problem 2

(a) Find the vertical asymptotes of the function.

$$y=\frac{x^2+1}{1-3x+2x^2}$$

(b) Confirm your answer to part (a) by graphing the function.

Problem 3 Use a graph to estimate the equations of all the vertical asymptotes of the curve

$$y=\ln\left(\cos^2 x\right) \qquad -\pi\le x\le\pi.$$

Then find the exact equations of these asymptotes.

Problem 4 Use the limit laws to evaluate the limit

$$\lim_{x\to 1}\left(\frac{1+2x}{1+4x^2+4x^4}\right)^3,$$

carefully justifying each step.

Problem 5 Consider two functions f and g, with the following graphs:

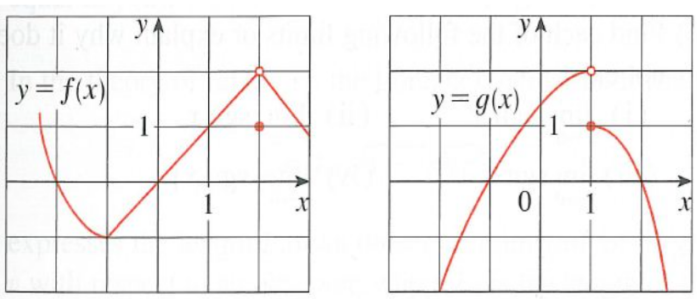

FIGURE 1. Graphs for Problem 5.

Use these graphs to evaluate the following limits. If the limit does not exist, explain why.

(a) $\lim_{x\to 2}[f(x)+g(x)]$	(b) $\lim_{x\to 1}[f(x)+g(x)]$	(c) $\lim_{x\to 0}[f(x)g(x)]$
(d) $\lim_{x\to -1}\frac{f(x)}{g(x)}$	(e) $\lim_{x\to 2}[x^3 f(x)]$	(f) $\lim_{x\to 1}\sqrt{3+f(x)}$

Problem 6 Evaluate the limit and justify each step by indicating the appropriate Limit Law(s).

$$\lim_{x\to 8}\left(1+\sqrt[3]{\frac{x^2}{2x-8}}\right)\left(x^3-7x^2-4\right).$$

Problem 7 If

$$f(x)=\begin{cases}\sqrt{x-2}, & \text{if } x>2\\ 4-2x, & \text{if } x<2\end{cases}$$

determine whether $\lim_{x\to 2} f(x)$ exists.

Problem 8 Let

$$f(x)=\begin{cases}4-\frac{1}{2}x, & \text{if } x<2\\ \sqrt{x+c}, & \text{if } x\geq 2\end{cases}$$

Find the value of c so that $\lim_{x\to 2} f(x)$ exists.

Problem 9

(a) What is wrong with the following equation?

$$\frac{x^2+x-6}{x+3}=x-2.$$

(b) Given your answer to part (a), explain why the following equation is correct.

$$\lim_{x\to -3}\frac{x^2+x-6}{x+3}=\lim_{x\to -3}(x-2).$$

Problem 10 Evaluate the limit, if it exists.

(a) $\lim_{t\to -3}\dfrac{t^2-9}{t^2+4t+3}$ (b) $\lim_{h\to 0}\dfrac{\sqrt{4+h}-2}{h}$ (c) $\lim_{x\to 6}\dfrac{\sqrt{x+3}-3}{x-6}$

(d) $\lim_{x\to -5}\dfrac{\frac{1}{5}+\frac{1}{x}}{5+x}$ (e) $\lim_{x\to 9}\dfrac{x^2-81}{\sqrt{x}-3}$ (f) $\lim_{t\to 0}\left(\dfrac{1}{t\sqrt{1+t}}-\dfrac{1}{t}\right)$

Problem 11 Do the following limits exist? If so, determine the limits.

(a) $\lim_{x\to 0^-}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$ (b) $\lim_{x\to 0^+}\left(\dfrac{1}{x}-\dfrac{1}{|x|}\right)$

Problem 12 Consider the function

$$F(x) = \frac{x^2 - 1}{|x - 1|}.$$

(a) Find $\lim_{x \to 1^+} F(x)$.

(b) Find $\lim_{x \to 1^-} F(x)$.

(c) Does $\lim_{x \to 1} F(x)$ exist?

(d) Sketch the graph of $F(x)$.

Problem 13 Use the squeeze theorem to show that

$$\lim_{x \to 0} \sqrt{x^3 + x^2} \sin \frac{\pi}{x} = 0.$$

Problem 14 Prove that

$$\lim_{x \to 0^+} \sqrt{x} \left[1 + \sin^2 \left(\frac{2\pi}{x} \right) \right] = 0.$$

Problem 15 Does the following limit exist? If so, determine the limit. If not, explain why not.

$$\lim_{x \to 3} (2x + |x - 3|).$$