NANYANG TECHNOLOGICAL UNIVERSITY

SPMS/DIVISION OF MATHEMATICAL SCIENCES

2020/21 Semester 1 MH1100 Calculus I Tutorial 4, Week 5

Your tutor will aim to discuss: Problem 1, 4, 5, 8, and 10

Problem 1 In this problem we will investigate the limit

$$\lim_{x\to 1}(2x+3)=5.$$

(a) Draw a graph of the function $f(x)=2x+3$ with the point $(1,5)$ marked.

(b) Add the lines $y=5+\epsilon$ and $y=5-\epsilon$ to your graph, with $\epsilon=2$. Using your graph, find a $\delta>0$ such that $|f(x)-5|<\epsilon$ whenever $0<|x-1|<\delta$.

(c) Repeat for the case $\epsilon=1.5$.

(d) Repeat for the case $\epsilon=1.0$.

(e) Repeat for the case $\epsilon=0.5$.

(f) Express $|f(x)-5|$ in terms of $|x-1|$.

(g) Use part (f) to give a general formula for $\delta(\epsilon)$ as a function of ϵ such that for every possible $\epsilon>0$, $|f(x)-5|<\epsilon$ whenever $0<|x-1|<\delta(\epsilon)$.

(h) Now write down a formal proof that $\lim_{x\to 1}(2x+3)=5$.

Problem 2 Use the ϵ, δ definition to prove that

$$\lim_{x\to -2}\left(\frac{1}{2}x+3\right)=2.$$

Problem 3 Use the ϵ, δ definition to prove that

$$\lim_{x\to 1}\frac{2+4x}{3}=2.$$

Problem 4 Use the ϵ, δ definition to prove that

$$\lim_{x\to a}x=a.$$

Problem 5 Use the ϵ, δ definition to prove that

$$\lim_{x\to a}c=c.$$

Problem 6 Use the ϵ, δ definition to prove that

$$\lim_{x\to 4}\frac{x^2-2x-8}{x-4}=6.$$

Problem 7 Use the ϵ, δ definition to prove that

$$\lim_{x\to 2}\left(x^2+2x-7\right)=1.$$

Problem 8 Consider the Heaviside function

$$H(t)=\begin{cases}1, & \text{if } t\geq 0\\ 0, & \text{if } t<0\end{cases}$$

Use the precise definition of a limit to prove that $\lim_{t\to 0}H(t)$ does not exist.

Problem 9 Let $a>0$ and n be a positive integer. Prove that

$$\lim_{x\to a}x^{\frac{1}{n}}=a^{\frac{1}{n}}.$$

Problem 10 Use the ϵ, δ definition to prove that

$$\lim_{x\to 0}|x|=0.$$