**MH1200 Linear Algebra I.**

Problem list for Week #1.

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This week's topics:

- Solving simple linear equations.
- Basics of the theory of vectors.

Note: for the questions on vectors, you may wish to use the graph paper drawn on the last page.

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## *Core problems:*

Problem 1:

Find the general solutions of the following linear equations

$$(i) \quad x + y + z = 1 \qquad\qquad (ii) \quad x - y + 3z = 5$$

What is the geometric interpretation of the solution set in each case?

Problem 2:

Find the general solution of the following linear system. (In other words find the points $(x, y, z)$ which solve both equations simultaneously.)

$$\begin{aligned} x + y + z &= 1 \\ x - y + 3z &= 5 \end{aligned}$$

We'll shortly learn some systematic procedures for solving systems like this, but it would be good to play with an example before we meet these procedures. Hint: you could use the 1st equation to eliminate one of the variables, say $x$, in the 2nd equation and then take one of the remaining variables, say $z$, as the free parameter. Then express $y$ and $x$ in terms of this free parameter to get an expression for the general solution.

What is the geometric interpretation of the solution set in this case?

Problem 3:

Give a parameterization for the set of solutions for the following linear equation:

$$x_1 + 2x_2 + 3x_3 + 4x_4 + 5x_5 = 6.$$

Problem 4:

Draw the following vectors:

1. $(-1, 2)$
2. $(-1, 2) + (3, -3)$
3. $(1, -2, 1)$
4. $(2, -1, -1)$

Problem 5:

Do there exist $a, b \in \mathbb{R}$ such that $(1, 1, 1) = a \cdot (1, -2, 1) + b \cdot (2, -1, -1)$?

Problem 6:

Let $\vec{u} = (1, 2)$ and $\vec{v} = (2, 1)$. Draw the vectors

1. $\frac{1}{2}\vec{u} + \frac{1}{2}\vec{v}$
2. $\frac{3}{4}\vec{u} + \frac{1}{4}\vec{v}$
3. $\frac{1}{4}\vec{u} + \frac{3}{4}\vec{v}$

Problem 7:

Let $\vec{u} = (1, 2)$ and $\vec{v} = (2, 1)$. Draw the following sets of vectors.

1. $\{c \cdot \vec{u} + (1 - c) \cdot \vec{v} : c \in \mathbb{R}, c \geq 0\}$.
2. $\{a\vec{u} + b\vec{v} : 0 \leq a \leq 1, 0 \leq b \leq 1\}$.
3. $\{a\vec{u} + b\vec{v} : a + b \leq 1\}$.

*More abstract or challenging problems:*

Problem 8:

Let $S \subset \mathbb{R}^n$ denote a set of vectors in $\mathbb{R}^n$.

(i) Let $\vec{w} \in \mathbb{R}^n$ be a fixed vector. Define a new set $S'$ from $S$ by:

$$S' = \{\vec{v} + \vec{w}, \vec{v} \in S\} \subset \mathbb{R}^n.$$

How is the set of geometric points in $\mathbb{R}^n$ corresponding to $S'$ geometrically related to the set of points in $\mathbb{R}^n$ corresponding to $S$?

(ii) Let $\lambda \in \mathbb{R}$ be a fixed real number and assume $\lambda > 0$. Define a new set $S''$ from $S$ by:

$$S'' = \{\lambda\vec{v}, \vec{v} \in S\} \subset \mathbb{R}^n.$$

How is the set of geometric points in $\mathbb{R}^n$ corresponding to $S''$ geometrically related to the set of points in $\mathbb{R}^n$ corresponding to $S$?