SPMS / Division of Mathematical Sciences

MH1300 Foundations of Mathematics
2017/2018 Semester 1

MID-TERM EXAM

9 October 2017 TIME ALLOWED: 50 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	10	3	15
2	10	4	15

Total:	50

TUTORIAL GROUP (Please tick)

	(T1) 1130–1230, TR3
	(T3) 1130–1230, TR6
	(T5) 1230–1330, TR3
	(T7) 1230–1330, TR6
	(T9) 1330–1430, TR3
	(T14) 1530–1630, TR5

	(T2) 1130–1230, TR5
	(T4) 1130–1230, TR7
	(T6) 1230–1330, TR5
	(T8) 1230–1330, TR7
	(T10) 1330–1430, TR5

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(10 marks)**

Derive the following logical equivalence **without** using truth tables:

$$(p \wedge q) \leftrightarrow p \;\equiv\; p \rightarrow q$$

You should use the list of logical equivalences in Theorem 2.1.1 of the lecture notes. You do not need to state the name of the logical equivalence at each step.

QUESTION 2 **(10 marks)**

Let $A = \{4, 8\}, B = \{2, 4\}$ and $C = \{1, 2, 4\}$. Determine if each of the following is true or false. Justify your answer.

(a) $\forall x \in A,\ \forall y \in B,\ \exists z \in C$ such that $x = yz$.

(b) $\exists x \in A$ such that $\forall y \in B,\ \forall z \in C,\ x = yz \rightarrow x = y + z$.

QUESTION 2 (Continued).

QUESTION 3. **(15 marks)**

Prove the following statements:

(a) Let x be an integer. If 3 does not divide $x^2 + 2$, then 3 divides x.

(b) Let a, b, c be integers. If a^2 does not divide bc, then either a does not divide b or a does not divide c.

QUESTION 3 (Continued).

QUESTION 4. **(15 marks)**

(a) Let $A = \{1, 2, 3, 4, 5, 6\}$ and $B = \{1, 2, 3, 4, 5, 6, 7\}$. For integers x and y, let $P(x)$ be the predicate "$7x + 4$ is odd" and let $Q(y)$ be the predicate "$5y + 9$ is odd". Let

$$S = \{(x, y) : x, y \in \mathbb{Z} \text{ and } \neg\,(P(x) \to Q(y))\}\,.$$

Write down the elements of $S \cap (A \times B)$ and $S \cap (B \times A)$.
(*Recall that $S \cap (A \times B)$ is the set of all elements belonging to both S and $A \times B$.*)

(b) Prove or disprove:

For all integers m and n, if $2m + 5n$ is odd then m and n are both odd.

QUESTION 4 (Continued).

END OF PAPER