SPMS / Division of Mathematical Sciences

MH1300 Foundations of Mathematics
2018/2019 Semester 1

MID-TERM EXAM

8 October 2018 TIME ALLOWED: 50 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	15	3	12
2	15	4	8

Total:	50

TUTORIAL GROUP (Please tick)

	(T5) 1230–1330, TR4
	(T7) 1230–1330, TR10
	(T10) 1330–1430, TR9
	(T12) 1330–1430, TR11
	(T14) 1530–1630, TR9

	(T6) 1230–1330, TR9
	(T9) 1330–1430, TR4
	(T11) 1330–1430, TR10
	(T13) 1530–1630, TR4

INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. You are allowed both sides of one A4 sized helpsheet.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(15 marks)**

Solve the following **without** using truth tables. You should use the list of logical equivalences in the lecture notes. You do not need to state the name of the logical equivalence at each step.

(a) Show that the following is a tautology:

$$(p \wedge q) \to (p \to q).$$

(b) Show that the following logical equivalence holds:

$$\neg(p \leftrightarrow q) \equiv \neg p \leftrightarrow q.$$

QUESTION 1 (Continued).

QUESTION 2 **(15 marks)**

Determine if each of the following is true or false. Justify your answer. The domain of x, y and z is the set of integers $\mathbb{Z}$.

(a) $\exists x\ \forall y,\ x < y^2$.

(b) $\forall x\ \forall y\ \exists z$ such that $z = (x + y)/2$.

(c) $\exists x\ \exists y$ such that $x^2 + y^2 = 6$.

QUESTION 2 (Continued).

QUESTION 3. **(12 marks)**

Prove or disprove the following statements:

(a) Let x be a real number. If x^3 is irrational then x is also irrational.

(b) Let n be an integer. If $3n+2$ is odd then $9n+5$ is even.

(c) Let n and m be integers. If $n+m^2$ is divisible by 3 then n or m is divisible by 3.

QUESTION 3 (Continued).

QUESTION 4. **(8 marks)**

Let a and b be real numbers. Show that the average of a and b is greater than a if and only if the average of a and b is less than b.

END OF PAPER