SPMS / Division of Mathematical Sciences
MH1300 Foundations of Mathematics
2019/2020 Semester 1

MID-TERM EXAM

7 October 2019 TIME ALLOWED: 50 MINUTES

NAME: **Matriculation Number:**

Question	Marks	Question	Marks
1	**14**	3	**14**
2	**12**	4	**10**

Total:	**50**

TUTORIAL GROUP (Please tick)

	(T1) 1130–1230, TR4 Ng Keng Meng		(T2) 1130–1230, TR9 Yuan Xinyue
	(T3) 1130–1230, TR10 Salah Mostafa		(T4) 1130–1230, TR11 Koh Heer Tern
	(T5) 1230–1330, TR4 Ng Keng Meng		(T6) 1230–1330, TR9 Yuan Xinyue
	(T9) 1330–1430, TR4 Koh Heer Tern		(T13) 1530–1630, TR9 Salah Mostafa

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INSTRUCTIONS TO CANDIDATES

1. This test paper contains **FOUR (4)** questions and comprises **EIGHT (8)** printed pages, including this cover page.
2. Answer **ALL** questions. This **IS NOT** an **OPEN BOOK** exam.
3. You are allowed both sides of one A4 sized helpsheet.
4. Candidates may use calculators. However, they should write down systematically the steps in the workings.

QUESTION 1. **(14 marks)**

Solve the following **without** using truth tables. You will **need to state** the logical equivalence used at each step.

(a) Determine if the following is a tautology, a contradiction, or neither: (5m)

$$((p \to \neg q) \land p) \land q.$$

(b) Determine which of the following is logically equivalent to $(p \lor q) \to r$: (9m)

(i) $(p \to r) \land (q \to r)$.

(ii) $(p \to r) \lor (q \to r)$.

(iii) $p \to (q \to r)$.

For each part (i), (ii) and (iii), if it is logically equivalent to $(p \lor q) \to r$, prove it without using truth tables. If it is not logically equivalent to $(p \lor q) \to r$, explain why not.

QUESTION 1 (Continued).

QUESTION 2 **(12 marks)**

Determine if each of the following is true or false. Justify your answer.

(a) For each positive integer a there is a positive integer b such that $\dfrac{1}{2b^2+b} < \dfrac{1}{ab^2}$.

(b) For each pair of integers x and y, there is an integer z such that $z^2 + 2xz - y^2 = 0$.

(c) There is some positive integer p such that $p^2 - 2$ is divisible by 3.

QUESTION 2 (Continued).

QUESTION 3. **(14 marks)**

(a) Let p, q be non-zero integers. If $p \mid q$ and $q \mid p$, show that $p = q$ or $p = -q$. (6m)

(b) Let x, y be real numbers. Prove from the definition of the absolute value function that $|xy| = |x||y|$. (8m)

QUESTION 3 (Continued).

QUESTION 4. **(10 marks)**

Prove that for any integer $n \geq 1$, $n^5 - n$ is divisible by 5.
Hint: *You may wish to first factorize* $n^5 - n$ *completely.*

END OF PAPER