MH1300 FOUNDATIONS OF MATHEMATICS

2020/21 Semester 1

Tutorial 1

Ex. 1.1.7b, 7d. Rewrite the following sentences less formally, without using variables. Determine, as best as you can, whether the statements are true or false.

b. There is a real number x such that $x^2 < x$.

d. For all real numbers a and b, $|a + b| \leq |a| + |b|$.

Ex. 1.1.9. Fill in the blanks and rewrite the given statement.

For all equations E, if E is quadratic then E has at most two real solutions.

a. All quadratic equations _____.

b. Every quadratic equation _____.

c. If an equation is quadratic, then it _____.

d. If E _____, then E _____.

e. For all quadratic equation E, _____.

Ex. 1.2.4.

a. Is $2 \in \{2\}$?

b. How many elements are in the set $\{2, 2, 2, 2\}$?

c. How many elements are in the set $\{0, \{0\}\}$?

d. Is $\{0\} \in \{\{0\}, \{1\}\}$?

e. Is $0 \in \{\{0\}, \{1\}\}$?

Ex. 1.2.7. Use the set-roster notation to indicate the elements in each of the following sets.

a. $S = \{n \in \mathbb{Z} |\ n = (-1)^k,\ \text{for some integer } k\}$.

b. $T = \{m \in \mathbb{Z} |\ m = 1 + (-1)^i,\ \text{for some integer } i\}$.

c. $U = \{r \in \mathbb{Z} |\ 2 \leq r \leq -2\}$.

d. $V = \{s \in \mathbb{Z} |\ s > 2 \text{ or } s < 3\}$.

e. $W = \{t \in \mathbb{Z} |\ 1 < t < -3\}$.

f. $X = \{u \in \mathbb{Z} |\ u \leq 4 \text{ or } u \geq 1\}$.

Ex. 1.2.9.
a. Is $3 \in \{1, 2, 3\}$?
b. Is $1 \subseteq \{1\}$?
c. Is $\{2\} \in \{1, 2\}$?
d. Is $\{3\} \in \{1, \{2\}, \{3\}\}$?
e. Is $1 \in \{1\}$?

f. Is $\{2\} \subseteq \{1, \{2\}, \{3\}\}$?
g. Is $\{1\} \subseteq \{1, 2\}$?
h. Is $1 \in \{\{1\}, 2\}$?
i. Is $\{1\} \subseteq \{1, \{2\}\}$?
j. Is $\{1\} \subseteq \{1\}$?

Ex. 1.2.10b, 10d. (Ordered pairs)
b. Is $(5, -5) = (-5, 5)$?
d. Is $\left(\frac{-2}{-4}, (-2)^3 \right) = \left(\frac{3}{6}, -8 \right)$?

Question E1. Are the following pairs of sets equal? Explain your answer.
a. $\{0, 1, 3, 5\}$ and $\{n \in \mathbb{Z} \mid n^2 - 1 \leq n\}$
b. $\{1, 1, 3\}$ and $\{3, 3, 1\}$
c. $\{x \in \mathbb{R} \mid 0 < x \leq 2\}$ and $\{1, 2\}$

Question E2. List out the elements of B in set-roster notation. Is $A \subseteq B$?

$$A = \{1, 2, 3, 4\}, \qquad C = \{5, 6, 7, 8\}, \qquad B = \{n \in A \mid n + m = 8 \text{ for some } m \in C\}$$

Question E3. Are the following true or false? Explain your answer.
a. $\{x\} \subseteq \{x\}$
b. $\{x\} \in \{x, \{x\}\}$
c. $\{x\} \in \{x\}$
d. $\{\{x\}\} \subsetneq \{x, \{x\}\}$

Question E4. Are the following true or false? Explain your answer.
a. $\{x, y\} \subseteq \{x, \{y\}\}$
b. $\{x\} \in \{x, \{x, y\}\}$
c. $\{\} \in \{x\}$
d. $\{y, \{x\}, \{y\}\} \subsetneq \{y, \{x\}, y, \{y, y\}\}$