MH1300 FOUNDATIONS OF MATHEMATICS

2020/21 Semester 1

Tutorial 2

Ex. 2.1.22, 24. Determine which of the pairs of statement forms are logically equivalent. Justify your answers using truth tables and include a few words of explanation.

22. $p \wedge (q \vee r)$ and $(p \wedge q) \vee (p \wedge r)$.

24. $(p \vee q) \vee (p \wedge r)$ and $(p \vee q) \wedge r$.

Ex. 2.1.28. Use De Morgan's laws to write a negation for the statement

"This computer program has a logical error in the first ten lines or it is being run with an incomplete data set."

Ex. 2.1.33. Assume x is a particular real number and use De Morgan's laws to write negations for the statement

$$-10 < x < 2.$$

Ex. 2.1.42. Use truth tables to establish which of the statement forms are tautologies and which are contradictions.

$$((\neg p \wedge q) \wedge (q \wedge r)) \wedge \neg q.$$

Ex. 2.1.49. Supply a reason for each step.

$$\begin{aligned}
(p \vee \neg q) \wedge (\neg p \vee \neg q) &\equiv (\neg q \vee p) \wedge (\neg q \vee \neg p) && \text{by } \underline{\text{ (a) }} \\
&\equiv \neg q \vee (p \wedge \neg p) && \text{by } \underline{\text{ (b) }} \\
&\equiv \neg q \vee \mathbf{F} && \text{by } \underline{\text{ (c) }} \\
&\equiv \neg q && \text{by } \underline{\text{ (d) }}
\end{aligned}$$

Therefore, $(p \vee \neg q) \wedge (\neg p \vee \neg q) \equiv \neg q$.

Ex. 2.1.52. Use the logical laws to verify the logical equivalence

$$\neg(p \vee \neg q) \vee (\neg p \wedge \neg q) \;\equiv\; \neg p.$$

Supply a reason for each step.

Ex. 2.2.12 (Modified). Apply Ex. 2.2.13a (below) to establish the logical equivalence

$$p \vee q \to r \;\equiv\; (p \to r) \wedge (q \to r).$$

Next, use it rewrite the following statement. (Assume that x represents a fixed real number.)

If $x > 2$ or $x < -2$, then $x^2 > 4$.

Ex. 2.2.13a. Use a truth table to verify the following logical equivalence. Include a few words of explanation with your answer.

$$p \to q \;\equiv\; \neg p \vee q.$$

Question E1. Are the following statement forms tautologies, contradictions, or neither? Justify your answer.

a. $p \wedge (p \vee \neg p)$
b. $\neg(p \vee \neg p) \vee p$
c. $\neg(p \wedge q) \vee p$