MH1300 FOUNDATIONS OF MATHEMATICS

2020/21 Semester 1

Tutorial 3

Ex. 2.2.20e, 21e, 22e. Write the negation, contrapositive, converse, and inverse for the following statement. (Assume that all variables represent fixed quantities or entities, as appropriate.)

"If x is nonnegative, then x is positive or x is 0."

Ex. 2.2.39. Definition: If r and s are statements, r **unless** s means if $\neg s$ then r.
Rewrite the statement in if-then form. "This door will not open unless a security code is entered."

Ex. 2.2.40. Rewrite the statement in if-then form. "Catching the 8:05 bus is a sufficient condition for my being on time for work."

Ex. 2.2.45. Note that "a sufficient condition for s is r" means r is a sufficient condition for s and that "a necessary condition for s is r " means r is a necessary condition for s. Rewrite the following statement in if-then form.

A necessary condition for this computer program to be correct is that it not produce error messages during translation.

Ex. 2.2.49. (a) Use the logical equivalences

$$p \to q \equiv \neg p \vee q \qquad \text{and} \qquad p \leftrightarrow q \equiv (\neg p \vee q) \wedge (\neg q \vee p)$$

to rewrite the given statement form without the symbol $\to$ and $\leftrightarrow$, and (b) use the logical equivalence

$$p \vee q \equiv \neg(\neg p \wedge \neg q)$$

to rewrite the given statement form using only $\wedge$ and $\neg$.

Given statement: $(p \to r) \leftrightarrow (q \to r)$.

Ex. 2.3.11. Use a truth table to determine whether the argument form is valid. Indicate which columns represent the premises and which represent the conclusion, and include a few words of explanation to support your answers.

$$p \to r$$
$$q \to r$$
$$\therefore p \vee q \to r.$$

Ex. 2.3.12. Use truth tables to show that the following forms of argument are invalid.

a.

$p \to q$
q
$\therefore\ p$
(converse error)

b.

$p \to q$
$\neg p$
$\therefore\ \neg q$
(inverse error)

Ex. 2.3.40. Sharky, a leader of the underworld, was killed by one of his own band of four henchmen. Detective Sharp interviewed the men and determined that all were lying except for one. He deduced who killed Sharky on the basis of the following statements:

a. Socko: Lefty killed Sharky.
b. Fats: Muscles didn't kill Sharky.
c. Lefty: Muscles was shooting craps with Socko when Sharky was knocked off.
d. Muscles: Lefty didn't kill Sharky.

Who did kill Sharky?

Ex. 2.3.43. Use the valid argument forms listed in the lecture notes to deduce the conclusion from the premises, giving a reason for each step. Assume all variables are statement variables.

a. $\neg p \to r \wedge \neg s$
b. $t \to s$
c. $u \to \neg p$
d. $\neg w$
e. $u \vee w$
f. $\therefore \neg t$

Ex. 3.1.7. Find the truth set of each predicate.

a. predicate: $6/d$ is an integer, domain: $\mathbb{Z}$.
b. predicate: $6/d$ is an integer, domain: $\mathbb{Z}^+$.
c. predicate: $1 \leq x^2 \leq 4$, domain: $\mathbb{R}$.
d. predicate: $1 \leq x^2 \leq 4$, domain: $\mathbb{Z}$.