MH1300 FOUNDATIONS OF MATHEMATICS

2020/21 Semester 1

Tutorial 7

Ex. 4.5.21. Prove the statement if it is true and find a counterexample if it is false.

For all odd integers n, $\lceil n/2 \rceil = (n+1)/2$.

Ex. 4.5.27. Prove the following statement.

For all real numbers x, if $x - \lfloor x \rfloor \geq 1/2$ then $\lfloor 2x \rfloor = 2\lfloor x \rfloor + 1$.

Ex. 4.6.14. Prove the statement by contradiction.

For all prime numbers a, b, and c, $a^2 + b^2 \neq c^2$.

Ex. 4.6.24. Prove the statement in two ways: (a) by contraposition and (b) by contradiction.

The reciprocal of any irrational number is irrational.

Note that the **reciprocal** of a nonzero real number x is $1/x$.

Ex. 4.7.18. The quotient-remainder theorem says not only that there exist quotients and remainders but also that the quotient and remainder of a division are unique. Prove the uniqueness. That is, prove that if a and d are integers with $d > 0$ and if q_1, r_1, q_2, and r_2 are integers such that

$$a = dq_1 + r_1 \qquad \text{where } 0 \leq r_1 < d$$

and

$$a = dq_2 + r_2 \qquad \text{where } 0 \leq r_2 < d,$$

then

$$q_1 = q_2 \qquad \text{and} \qquad r_1 = r_2.$$

Ex. 4.7.23. Prove that $\sqrt{2} + \sqrt{3}$ is irrational.

Ex. 4.7.24. Prove that $\log_5(2)$ is irrational. (Hint: Use the unique factorization of integers theorem.)

Ex. 4.7.28. An alternative proof of the infinitude of the prime numbers begin as follows:

Proof: Suppose there are only finitely many prime numbers. Then one is the largest. Call it p. Let $M = p! + 1$. We will show that there is a prime number q such that $q > p$.

Complete this proof. ($p! = p(p-1)\cdots 3 \cdot 2 \cdot 1$.)