MH1300 FOUNDATIONS OF MATHEMATICS

2020/21 Semester 1

Tutorial 8

Ex. 5.1.56. Transform the sum by making the change of variable $j = i - 1$.

$$\sum_{i=3}^{n} \frac{i}{i+n-1}.$$

Ex. 5.1.77.

a. Prove that $n! + 2$ is divisible by 2, for all integers $n \geq 2$.
b. Prove that $n! + k$ is divisible by k, for all integers $n \geq 2$ and $k = 2, 3, \ldots, n$.
c. Given any integer $m \geq 2$, is it possible to find a sequence of $m - 1$ consecutive positive integers none of which is prime? Explain your answer.

Ex. 5.2.14. Prove the following statement by mathematical induction.

$$\sum_{i=1}^{n+1} i \cdot 2^i \;=\; n \cdot 2^{n+2} + 2, \quad \text{for all integers } n \geq 0.$$

Ex. 5.3.11. Prove the following statement by mathematical induction.

$$3^{2n} - 1 \text{ is divisible by 8, for each integer } n \geq 0.$$

Ex. 5.3.12. Prove the following statement by mathematical induction.

$$7^n - 2^n \text{ is divisible by 5, for each integer } n \geq 0.$$

Ex. 5.3.20. Prove the following statement by mathematical induction.

$$2^n < (n+2)!, \text{ for all integers } n \geq 0.$$

Ex. 5.3.22. Prove the following statement by mathematical induction.

$$1 + nx \leq (1+x)^n, \text{ for all real numbers } x > -1 \text{ and integers } n \geq 2.$$